

## RICERCHE

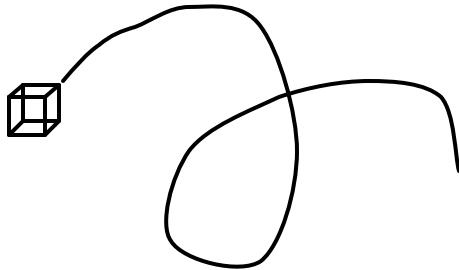
$$\left(u' \cdot v'\right)_m \quad \text{oppure} \quad \mathbf{r} \cdot \left(u' \cdot v'\right)_m$$

1. Sperimentali

2. Teoriche

**Prandtl:** come per teoria cinetica dei gas  
*percorso di mescolamento turbolento*

Hp.



*conserva:* la sua individualità  
la sua q.ta di moto

**Altre ipotesi di altri AA** conducono a risultati anch'essi  
soddisfacenti

## Analisi dimensionale

### A) Moto uniforme

$$t_0 = f(D, S, V, \mathbf{m}, \mathbf{r}) \quad \text{condotto circolare}$$

$$t_0 = f(R, \mathbf{a}, S, V, \mathbf{m}, \mathbf{r}) \quad \text{condotto di forma qualunque}$$

└───▶ forma

### Moto unif. laminare

$$t_0 = f_1(D, V, \mathbf{m}) \quad \left( \begin{array}{l} \text{controllo} \\ \text{speriment.} \end{array} \right)$$

S coperto da pellicola aderente parete

$\rho$  non vi sono scambi di quantità di moto

$D, V, \mu$  grandezze fondamentali

$$t_0 = k \cdot D^a \cdot V^b \cdot \mathbf{m}^g$$

$$t_0 [F \cdot L^{-2}] \quad D [L] \quad V [L \cdot T^{-1}] \quad \mathbf{m} [F \cdot T \cdot L^{-2}]$$

$$F \cdot L^{-2} = L^a \cdot L^b \cdot T^{-b} \cdot F^g \cdot T^g \cdot L^{-2 \cdot g}$$

$$\begin{array}{l} L \rightarrow \\ T \rightarrow \\ F \rightarrow \end{array} \left\{ \begin{array}{l} \mathbf{a} + \mathbf{b} - 2 \cdot \mathbf{g} = -2 \\ -\mathbf{b} + \mathbf{g} = 0 \\ \mathbf{g} = 1 \end{array} \right. \quad \begin{array}{l} \mathbf{a} = -1 \\ \mathbf{b} = 1 \\ \mathbf{g} = 1 \end{array}$$

$$t_0 = k \cdot \frac{m \cdot V}{D}$$

$$t_0 = g \cdot \frac{D}{4} \cdot J \rightarrow J = 4 \cdot k \cdot \frac{m}{g} \cdot \frac{V}{D^2}$$

$$J = 32 \cdot \frac{m}{g} \cdot \frac{V}{D^2} \quad \text{teorico} \quad (k=8)$$

$$t_0 = 8 \cdot \frac{m \cdot V}{D}$$

**BASTA 1 PROVA** : noti  $\mu$ ,  $V$ ,  $D$ ,  $J \rightarrow k$

### **B) Moto unif. puramente turbolento**

Hp.  $s \propto D$  oppure  $s = 0$

$$t_0 = f_2(D, V, \mathbf{r})$$

$$t_0 = l_1 \cdot D^a \cdot V^b \cdot \mathbf{r}^g$$

$$D[L] \quad V[L \cdot T^{-1}] \quad \mathbf{r}[F \cdot L^{-4} \cdot T^2] \quad t[F \cdot L^{-2}]$$

$$F \cdot L^{-2} = L^a \cdot L^b \cdot T^{-b} \cdot F^g \cdot L^{-4g} \cdot T^{2g}$$

$$\begin{cases} \mathbf{a} + \mathbf{b} - 4 \cdot \mathbf{g} = -2 \\ 2 \cdot \mathbf{g} - \mathbf{b} = 0 \\ \mathbf{g} = 1 \end{cases} \quad \begin{cases} \mathbf{a} = 0 \\ \mathbf{b} = 2 \\ \mathbf{g} = 1 \end{cases}$$

$$t_0 = l_1 \cdot r \cdot V^2$$

- basterebbe 1 sola prova per  $\lambda_1$
- se c'è  $S \rightarrow l_1 = l_1 \cdot \left(\frac{S}{D}\right)$ : basta 1 serie di prove

$$t_0 = g \cdot \frac{D}{4} \cdot J$$

$$J = \frac{4 \cdot l_1 \cdot r \cdot V^2}{g \cdot D} = 4 \cdot \frac{l_1}{g} \cdot \frac{V^2}{D}$$

$$l_1 = \frac{t_0}{r \cdot V^2}$$

- n° indice di resistenza

- resistenza ridotta

### DARCY – WEISBACH

$$J = l \cdot \frac{V^2}{2 \cdot g \cdot D} = 8 \cdot l_1 \cdot \frac{V^2}{2 \cdot g \cdot D} \quad \text{formula sperimentale}$$

$$[l = 8 \cdot l_1]$$

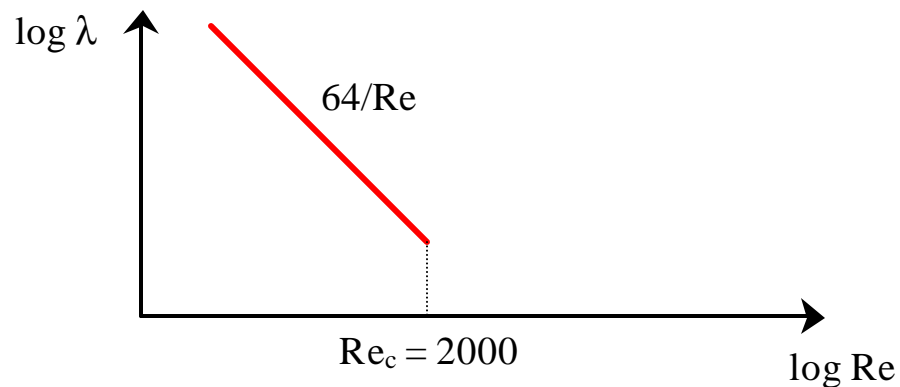
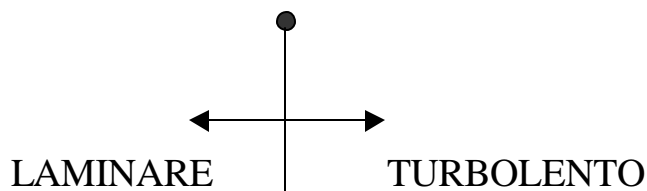
## Numero di Reynolds

$$\frac{t_0 \text{ TURB.PUR.}}{t_0 \text{ LAM.}} = \frac{l_1 \cdot r \cdot V^2}{k \cdot m \cdot \frac{V}{D}} = \text{cost} \cdot \frac{r \cdot V \cdot D}{m}$$

$\downarrow$   $\qquad \qquad \qquad \downarrow$   
 n° puro  $\qquad \qquad \infty$   $\qquad \qquad \qquad$  n° puro

$$\text{Re} = \frac{r \cdot V \cdot D}{m} \quad \text{indice del grado di turbolenza}$$

$\exists \quad \text{Re}_c \cong 2000$       valore critico ( stato critico )



$$l = 8 \cdot l_1 = 8 \cdot \frac{t_0}{r \cdot V^2} = \frac{8}{r \cdot V^2} \cdot \frac{8 \cdot m \cdot V}{D} = \frac{64}{\text{Re}} = 64 \cdot \text{Re}^{-1}$$

**TUBI LISCI** ( moto uniforme in zona di transizione )

$$S = 0$$

$$t_0 = f_3(D, V, r, m)$$

$$m = \frac{r \cdot V \cdot D}{\text{Re}}$$

$$t_0 = f_4(D, V, r, \text{Re})$$

$$t_0 = I_1 \cdot (\text{Re}) \cdot r \cdot V^2$$

*occorre 1 serie di esperienze*

**Blasius** ( $\text{Re}_c < \text{Re} \leq 10^5$  e oltre )

$$I = 0,316 \cdot \text{Re}^{-0.25}$$

$$J = I \cdot \frac{V^2}{2 \cdot g \cdot D} = \frac{0.316}{2 \cdot g} \cdot \frac{m^{0.25}}{r^{0.25} \cdot V^{0.25} \cdot D^{0.25}} \cdot \frac{V^2}{D} = \cos t \cdot \frac{V^{1.75}}{D^{1.25}}$$

↑  
dipende dal  
fluido

**Prandtl – V.Karman**

$$\frac{1}{\sqrt{I}} = -2 \cdot \log \frac{2.51}{\text{Re} \cdot \sqrt{I}}$$

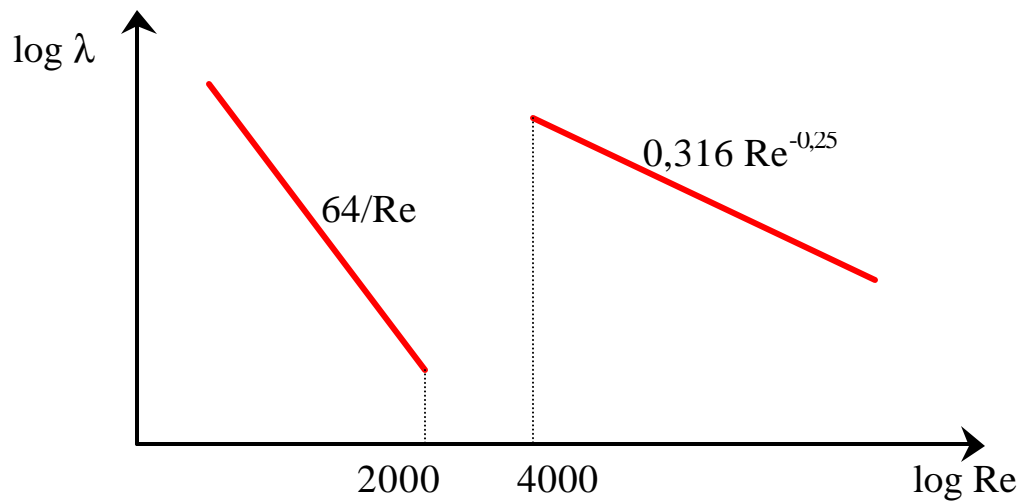
" tipo di moto

$$J = k \cdot \frac{V^m}{D^n} \quad m+n=3$$

Hp. se  $I = a \cdot \text{Re}^{-b}$  (monomia)

$$J = I \cdot \frac{V^2}{2 \cdot g \cdot D} = \cos t \cdot \frac{V^{2-b}}{D^{1+b}}$$

$$m + n = 2 - b + 1 + b = 3 \quad \text{c.v.d.}$$



L'espressione di Prandtl-V.Karman

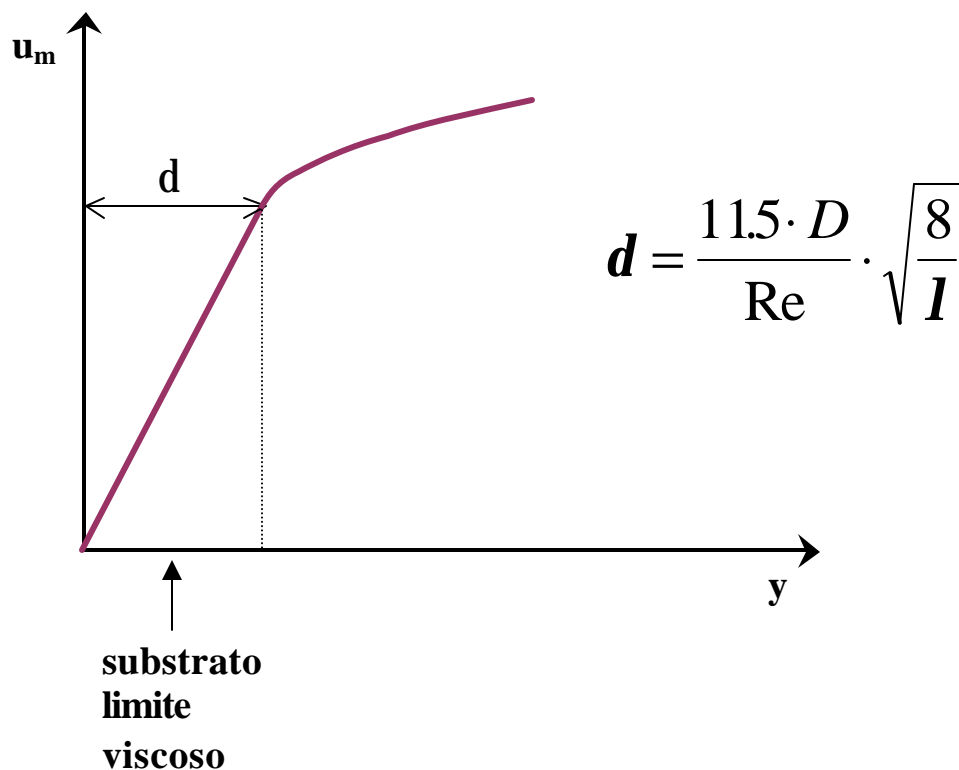
$$\frac{1}{\sqrt{I}} = -2 \cdot \log \frac{2.51}{\text{Re} \cdot \sqrt{I}}$$

deriva da :

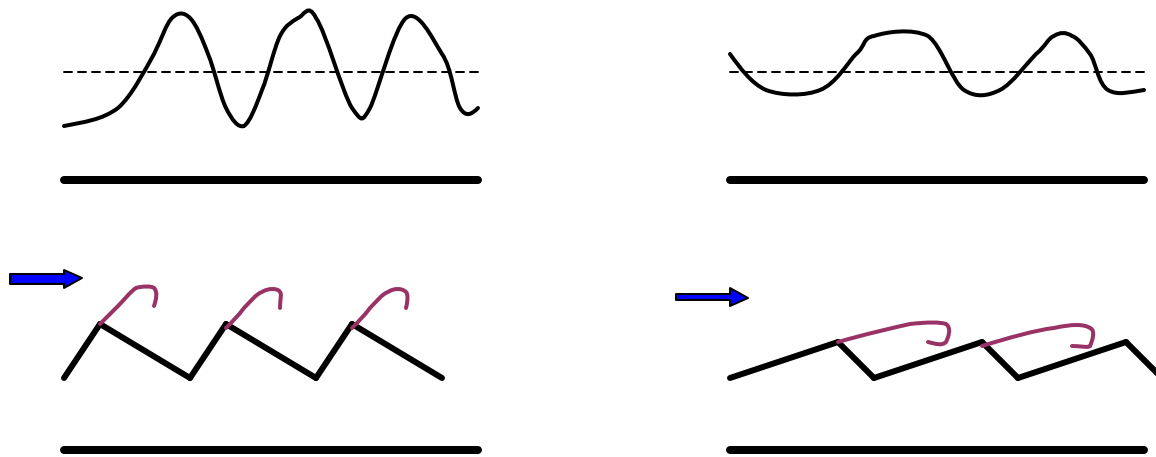
- Hp. percorso di mescolamento turbolento (Prandtl)
- Hp. similitudine dei diagrammi di velocità in tubi lisci

Si introduce

$$u^* = \sqrt{\frac{t_0}{r}} = \sqrt{\frac{I_1 \cdot r \cdot V^2}{r}} = V \cdot \sqrt{\frac{I}{8}} \quad \text{velocità di attrito}$$

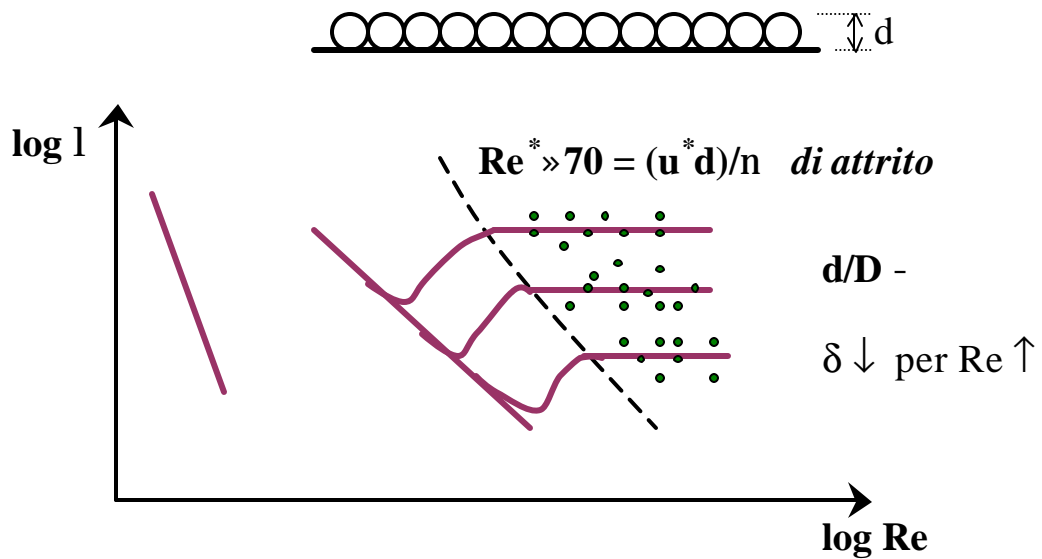


Moto uniforme **TUBI SCABRI**



per definire  $S$  non basta una lunghezza

**Nikuradse (1930)**

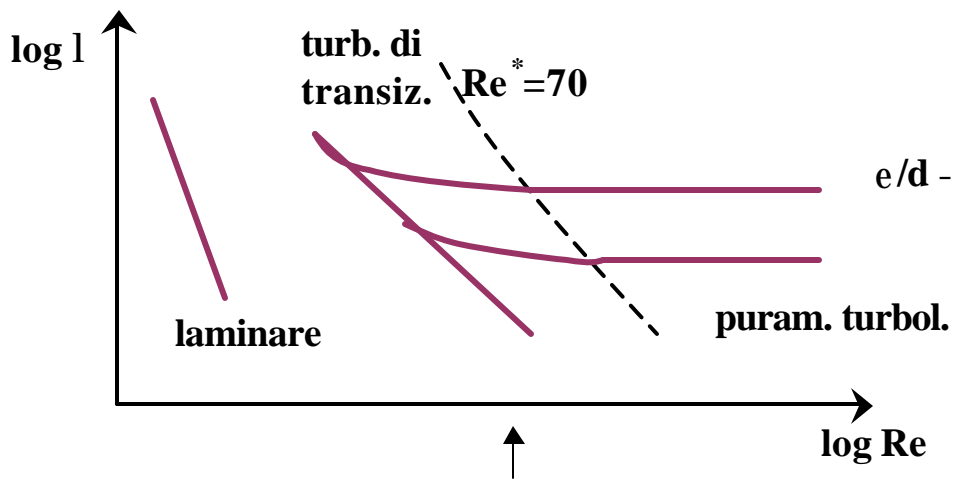


$l = \text{cost}$  MOTO PURAMENTE TURBOLENTO

**Prandtl-V.Kàrmàn per tubi scabri**

$$\frac{1}{\sqrt{I}} = -2 \cdot \log\left(\frac{1}{3.71} \cdot \frac{d}{D}\right)$$

## Moto unif. **TUBI SCABRI REALI**



### **ABACO DI MOODY**

Le asperità escono gradualmente dallo strato limite viscoso

### **COLEBROOK e WHITE**

$$\frac{1}{\sqrt{f}} = -2 \cdot \log \left( \frac{2.51}{Re \cdot \sqrt{f}} + \frac{1}{3.71} \cdot \frac{e}{D} \right)$$

va bene nella zona turb. di transizione perché si raccorda con tubi lisci ( $[e/(3,71 D)]$ ) trascurabile rispetto a  $[2,51/(Re \cdot \sqrt{f})]$ , ma vale anche per il moto puramente turbolento ( $[2,51/(Re \cdot \sqrt{f})]$  trascurabile rispetto a  $[e/(3,71 D)]$ )

**MA**  $f$  implicito

## VERIFICA

1. Dati :  $Q, D, \varepsilon$  ?  $J$

↓  
Re → Moody  $\lambda$  →  $J$

MOODY

2. Dati :  $J, D, \varepsilon$  ?  $V, Q$

↓

$$\text{Re} \cdot \sqrt{I} = \text{Re} \cdot \frac{\sqrt{2 \cdot g \cdot D \cdot J}}{V} = \frac{r \cdot D}{m} \cdot \underbrace{\sqrt{2 \cdot g \cdot D \cdot J}}_{\text{noto}}$$

COLEBR.  $\lambda \Rightarrow V = \sqrt{\frac{2 \cdot g \cdot D \cdot J}{I}} \Rightarrow Q$

## PROGETTO

Dati :  $Q, J, \varepsilon$  ?  $D$

$$D = l \cdot \frac{V^2}{2 \cdot g \cdot J} \Rightarrow D^5 = l \cdot \frac{Q^2}{2 \cdot g \cdot J \cdot \left(\frac{p}{4}\right)^2}$$

MOODY :

$$D_1 \rightarrow \frac{e}{D_1}, \text{Re}_1 \rightarrow I' \rightarrow D_2 = \sqrt[5]{I' \cdot \frac{Q^2 \cdot 16}{2 \cdot g \cdot J \cdot p^2}}$$

↑  
ecc.

## Formule esplicite ( Colebrook )

### 1. Cozzo

$$I = 0.021 \cdot \frac{g \cdot P^2}{8} \cdot \left\{ -\log \left[ \frac{5.1 \cdot (n \cdot D)^{0.91}}{Q^{0.091}} + \frac{e}{3.71 \cdot D} \right] \right\}^{-2}$$

$\lambda < \pm 4 \%$  ma normalmente + piccoli

### 2. Citrini

$$\left\{ \begin{aligned} I &= I_{\infty} \cdot \left( 1 + \frac{4}{\text{Re} \cdot \frac{e}{D}} \right)^2 \cong I_{\infty} \cdot \left( 1 + \frac{8}{\text{Re} \cdot \frac{e}{D}} \right) \\ I_{\infty} &= \frac{1}{4} \cdot \left( \log 3.71 \cdot \frac{D}{e} \right)^{-2} \end{aligned} \right.$$

## FORMULE PRATICHE (Empiriche)

### 1. Più antica COUPLET (Versailles)

$$\frac{D \cdot J}{V^2} = \text{cost} \quad \text{Moto pur. turbolento}$$

↑  
↘ assoluta ?!

### 2. PRONY

$$\frac{D \cdot J}{V^2} = a + \frac{b}{V} \quad \text{Moto turb. di transizione}$$

### 3. DARCY

$$\frac{D \cdot J}{V^2} = a + \frac{b}{D}$$

- moto puramente turbolento
- scabrezza relativa  $\frac{b}{D} \left( \equiv \frac{e}{D} \right)$
- tubi in ghisa
- a e b doppi dopo esercizio

$$J = \mathbf{b} \cdot \frac{Q^2}{D^5}$$

$$\mathbf{b} = a_1 + \frac{b}{D} \quad D \leq 500mm$$

#### 4. CHEZY

$$J = \frac{V^2}{C^2 \cdot R}$$

$$C = \sqrt{\frac{8 \cdot g}{I}}$$

*coefficiente di scabrezza*

- C non è n° puro
- Per un assegnato tubo si ha che :  $C = \text{cost} (\equiv) \lambda = \text{cost} \rightarrow$  moto pur. turb.  
 $C_\infty (\equiv \lambda_\infty)$  non dipende dal fluido

- Bazin

$$C = \frac{87}{1 + \frac{g}{\sqrt{R}}}$$

← indice di scabrezza

- Kutter

$$C = \frac{100}{1 + \frac{m}{\sqrt{R}}}$$

- Strickler

$$C = c \cdot R^{\frac{1}{6}} \quad c \uparrow, e \downarrow$$

$$J = \frac{V^2}{c^2 \cdot R^{\frac{4}{3}}}$$

- Manning

$$n = \frac{1}{c}$$

## 5. CONTESSINI

condotte in acciaio

$$J = \mathbf{b} \cdot \frac{Q^2}{D^a}$$

$$\beta = 0.0012 \quad \alpha = 5.26 \quad \text{tubi nuovi}$$

$$\beta = 0.0020 \quad \alpha = 5.44 \quad \text{tubi usati}$$

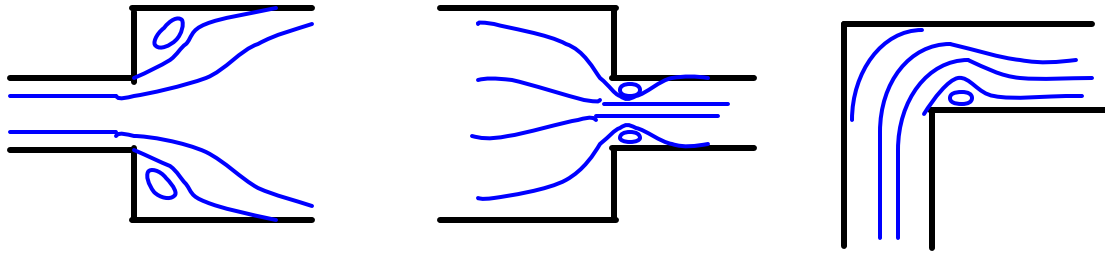
UTILE PER DIMENSIONAMENTO

## 6. MONOMIE

$$J = a \cdot \frac{Q^b}{D^c} \quad 1.75 \leq b \leq 2$$

**valgono solo per il fluido sperimentato**

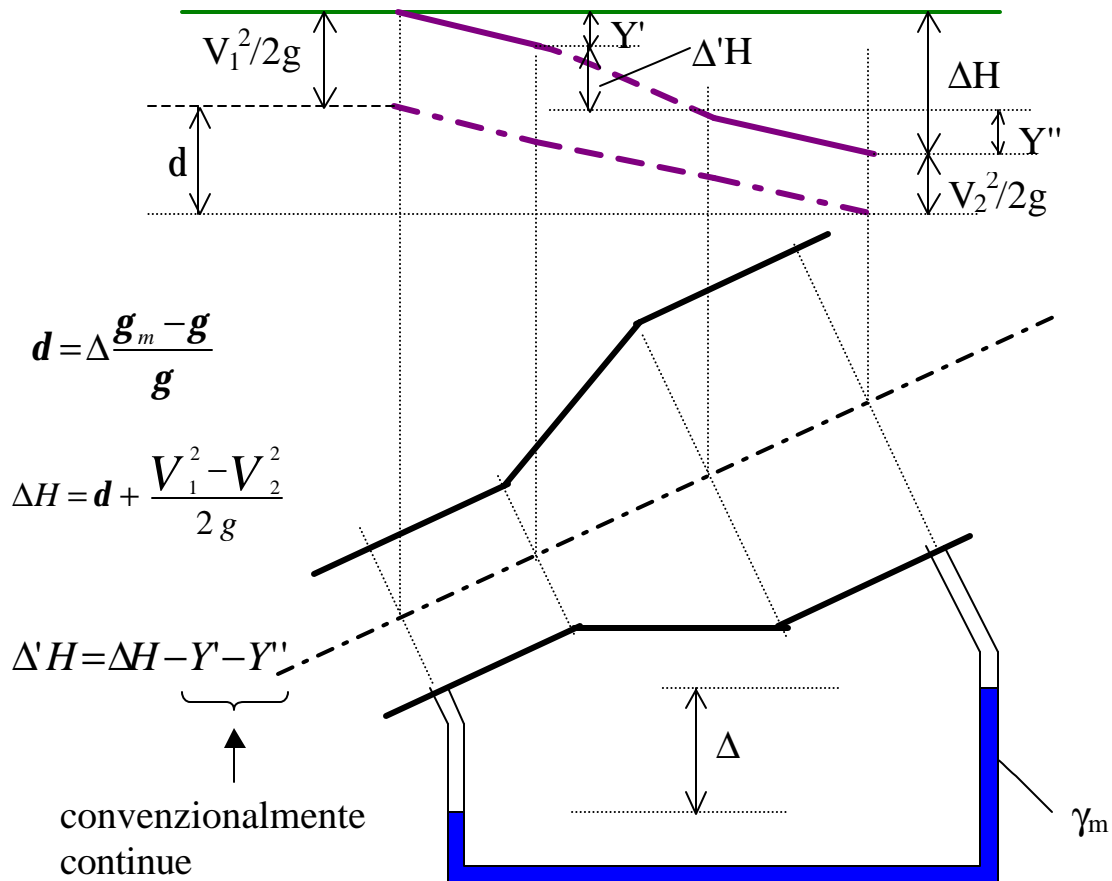
# PERDITE DI CARICO LOCALIZZATE



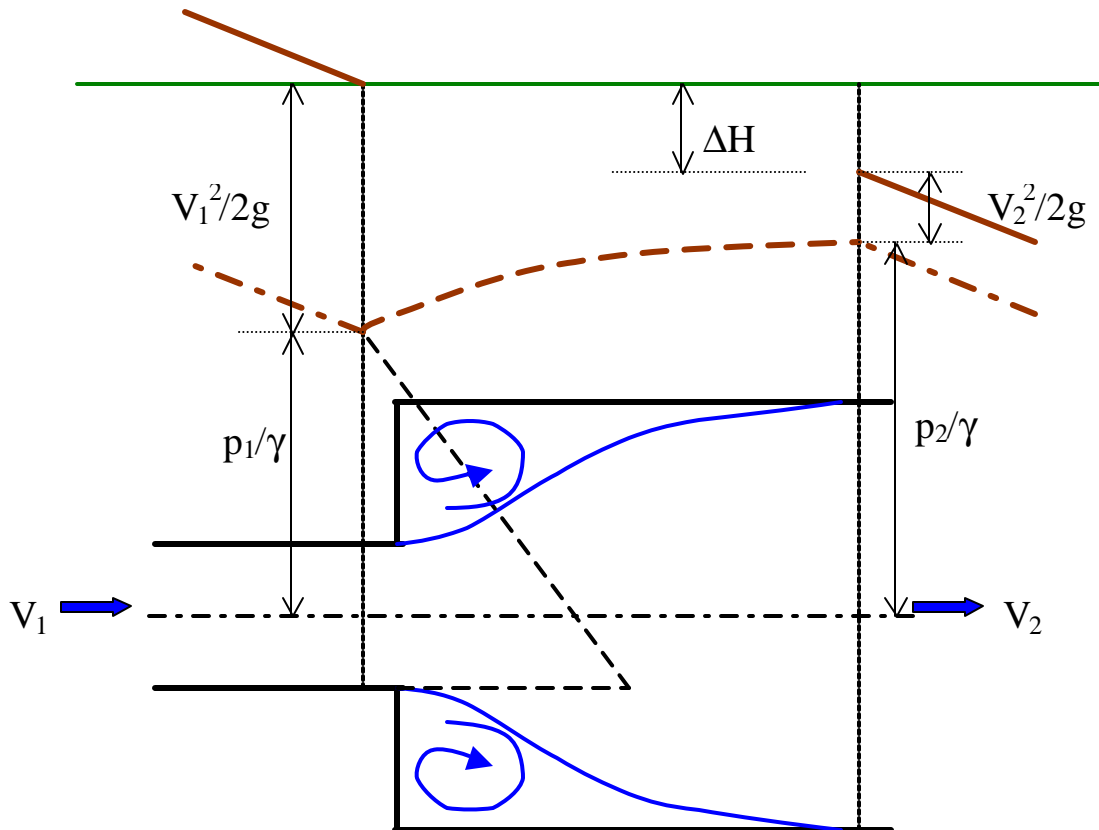
- linee carichi totali e piezometriche sono solo "linee di raccordo"

- Come moto puram. turbol. 
$$\Delta H = n \cdot \frac{V^2}{2 \cdot g}$$

$n = \text{cost}$  [  $n = n(\text{Re})$  solo per (  $V$  basse )  $\text{Re}$  bassi ]



## Brusco allargamento



- Hp.** 1. Resistenza delle pareti trascurabile:  $v$  variabile come verso, tronco breve  
 2. Arbitraria:  $p$  idrostatica sulla corona circolare

Proiezione dell'eq. globale nella direzione del moto :

$$\begin{cases} p_1 \cdot A_1 + p_1 \cdot (A_2 - A_1) + \mathbf{r} \cdot Q \cdot V_1 = p_2 \cdot A_2 + \mathbf{r} \cdot Q \cdot V_2 \\ Q = A_2 \cdot V_2 \end{cases}$$

$$A_2 \cdot (p_2 - p_1) = \mathbf{r} \cdot A_2 \cdot V_2 \cdot (V_1 - V_2)$$

> 0 perché  $V_2 < V_1$   
 piezometrica aumenta

Se condotta inclinata :

$$\left( \frac{p_2}{g} + z_2 \right) > \left( \frac{p_1}{g} + z_1 \right)$$

$$\left\{ \begin{array}{l} \Delta H = \left( z_1 + \frac{p_1}{g} + \mathbf{a}_1 \cdot \frac{V_1^2}{2 \cdot g} \right) - \left( z_2 + \frac{p_2}{g} + \mathbf{a}_2 \cdot \frac{V_2^2}{2 \cdot g} \right) \\ z_1 = z_2 \\ \mathbf{a}_1 \cong \mathbf{a}_2 \cong 1 \\ p_2 - p_1 = \mathbf{r} \cdot V_2 \cdot (V_1 - V_2) \end{array} \right.$$

$$\rightarrow \Delta H = \frac{1}{2 \cdot g} \cdot (V_1^2 - V_2^2 + 2 \cdot V_2^2 - 2 \cdot V_1 \cdot V_2)$$

$$\Delta H = \frac{(V_1 - V_2)^2}{2g}$$

**formula di BORDA**

↳ altezza cinetica della velocità perduta

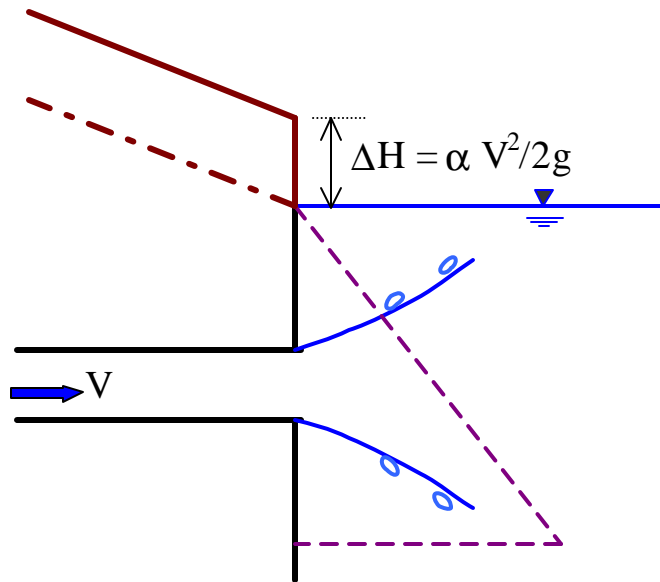
**Hp.1** significa :  $t = 0$  ® **fluido perfetto**

La perdita  $\Delta H$  è perdita di energia meccanica (cinetica)

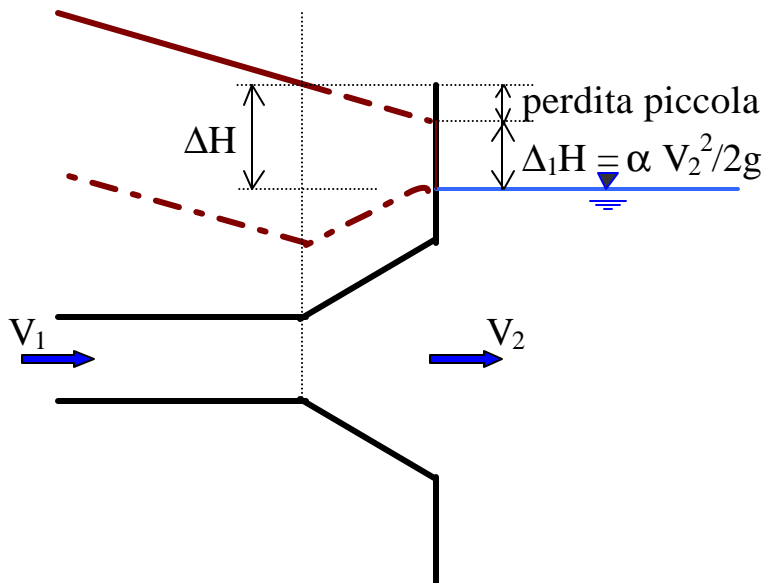
UTILE:  $\mathbf{a} \cdot \frac{V^2}{2 \cdot g}$ . Va in agitazione che si dissipa solo con

l'intervento di  $\mu$

## Perdita di sbocco

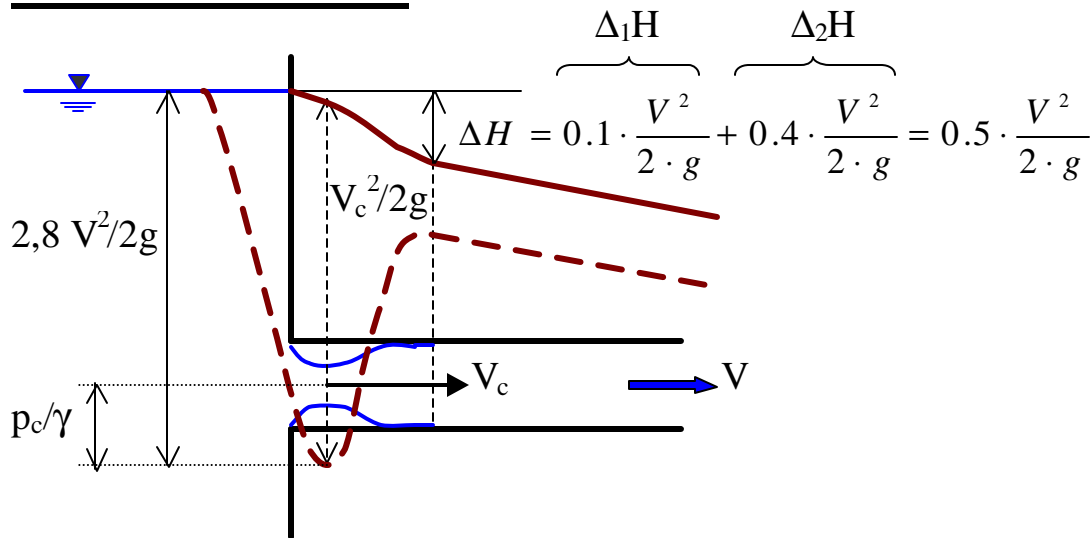


$$\Delta H = \frac{(V_1 - V_2)^2}{2 \cdot g} = \frac{(V_1 - 0)^2}{2 \cdot g} = \frac{V_1^2}{2 \cdot g} \cdot (\mathbf{a})$$



$$\Delta H = \underbrace{\mathbf{a} \cdot \frac{V_2^2}{2 \cdot g}}_{\Delta_1 H} + \textit{perdita nel divergente}$$

## Perdita di imbocco



Esperienza :  $C_c = 0.61$                        $C_v = 0.98$

$$\Delta H = \Delta_1H + \Delta_2H$$

$$V_t(\text{torricelliana}) \quad V_c = C_v \cdot V_t$$

$$A_c \cdot V_c = A_c \cdot V_c \cdot C_c = A \cdot V \Rightarrow V_c = \frac{V}{C_c}$$

$$\Delta_1H = \frac{V_t^2}{2 \cdot g} - \frac{V_c^2}{2 \cdot g} = \frac{V_c^2}{2 \cdot g} \cdot \left( \frac{1}{C_v^2} - 1 \right) = \frac{V_c^2}{2 \cdot g} \cdot \frac{1 - C_v^2}{C_v^2 (\cong 1)} =$$

$$= \frac{V_c^2}{2 \cdot g} \cdot (1 - C_v^2) =$$

$$= \frac{V^2}{2 \cdot g} \cdot \frac{1 - C_v^2}{C_c^2} \cong 0.1 \cdot \frac{V^2}{2 \cdot g}$$

$$\Delta_2H = \frac{(V_c - V)^2}{2 \cdot g} (\text{BORDA}) = \frac{1}{2 \cdot g} \cdot \left( \frac{V}{C_c} - V \right)^2 =$$

$$= \frac{V^2}{2 \cdot g} \cdot \left( \frac{1}{C_c} - 1 \right)^2 \cong 0.4 \cdot \frac{V^2}{2 \cdot g}$$

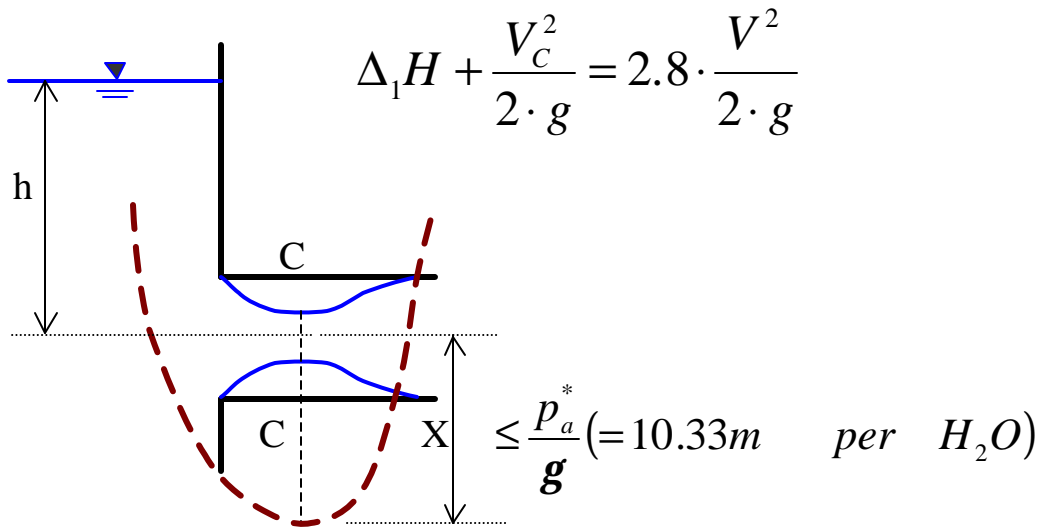
$$\Delta H = \Delta H_1 + \Delta H_2$$

$$\Delta H = 0.5 \cdot \frac{V^2}{2 \cdot g}$$

Piezometrica (da linea CT)

$$\frac{V_c^2}{2g} = \frac{1}{C_c^2} \frac{V^2}{2g} = 2.7 \frac{V^2}{2g}$$

Piezometrica (da pelo libero)



Se da calcolo  $X > \frac{p_a^*}{g}$

→  $X = \frac{p_a^*}{g}$  e CC sez. di controllo

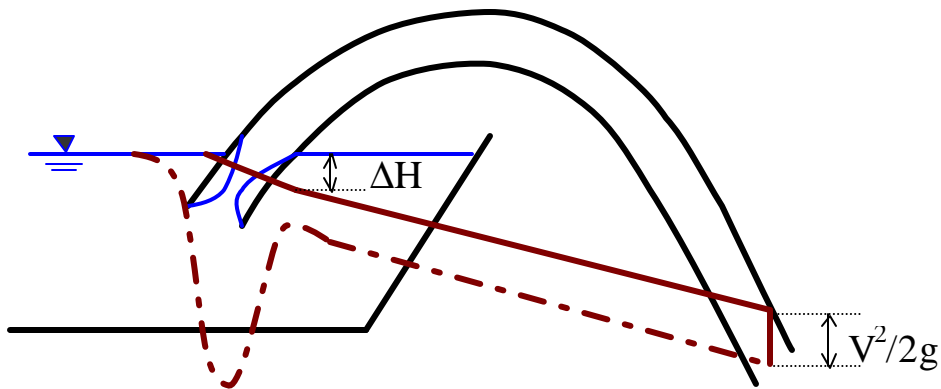
$$V_{C_{\max}} = \sqrt{2 \cdot g \cdot \left( h + \frac{p_a^*}{g} \right)} \quad (\text{trascuro } \Delta_1 H)$$

$$\text{H}_2\text{O} \rightarrow Q_{\max} = C_c \cdot A \cdot \sqrt{2 \cdot g \cdot (h + 10.33)}$$

in realtà  $\frac{p_a^* - p_v^*}{g} \cong 10.33 - 0.20m$

$$Q_{\max} \cong C_c \cdot A \cdot \sqrt{2 \cdot g \cdot (h + 10.13)}$$

### Altra perdita d'imbocco



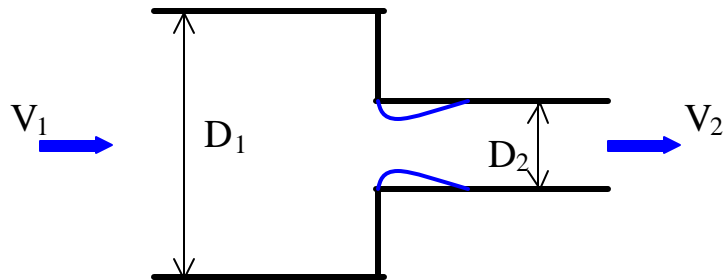
$$C_c = \text{min} \cong 0.5$$

$$\Delta_1 H = \frac{V^2}{2 \cdot g} \cdot \frac{1 - C_v^2}{C_c^2} = 0.16 \cdot \frac{V^2}{2 \cdot g}$$

$$\Delta_2 H = \frac{V^2}{2 \cdot g} \cdot \left( \frac{1}{C_c} - 1 \right)^2 = \frac{V^2}{2 \cdot g}$$

$$\Delta H = 1.16 \cdot \frac{V^2}{2 \cdot g}$$

## Perdita per brusco restringimento

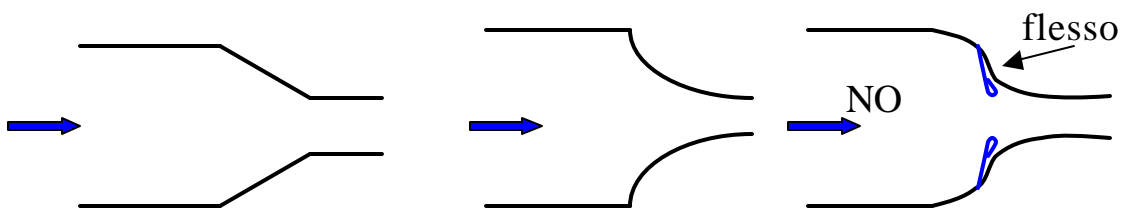


Se  $\frac{D_1}{D_2} \geq 2$   $\Delta H = 0.5 \cdot \frac{V_2^2}{2 \cdot g}$

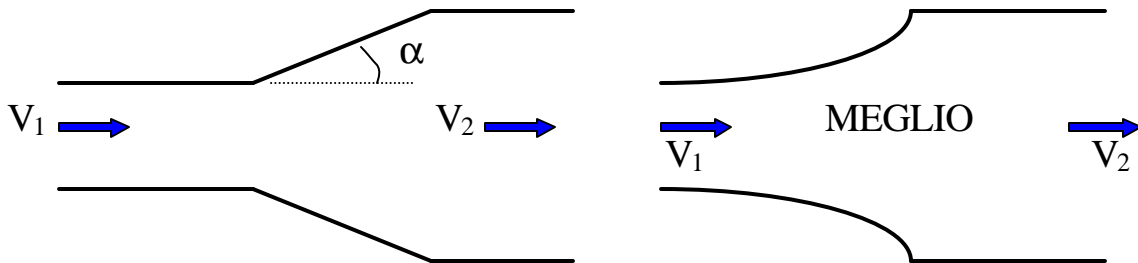
Se  $\frac{A_1}{A_2} = 2 \cdot \left( \frac{D_1}{D_2} = \sqrt{2} \right)$   $\Delta H = 0.3 \cdot \frac{V_2^2}{2 \cdot g}$

$$\Delta H = n \cdot \frac{V_2^2}{2 \cdot g}$$

## Convergenti



# DIVERGENTI



## Gibson

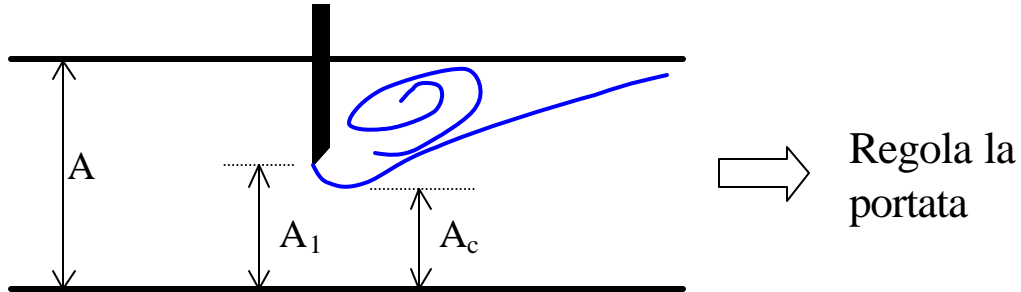
$$\Delta H = m(\mathbf{a}) \cdot \frac{(V_1 - V_2)^2}{2 \cdot g}$$

$m = \text{MIN}$       per  $\alpha = 6^\circ$   
 $m = \text{MAX} = 1.2$       per  $\alpha = 65^\circ$       } dipende dalle perdite continue

Escande : creando depressioni elimina il distacco di vena e diminuisce  $\Delta H$ .

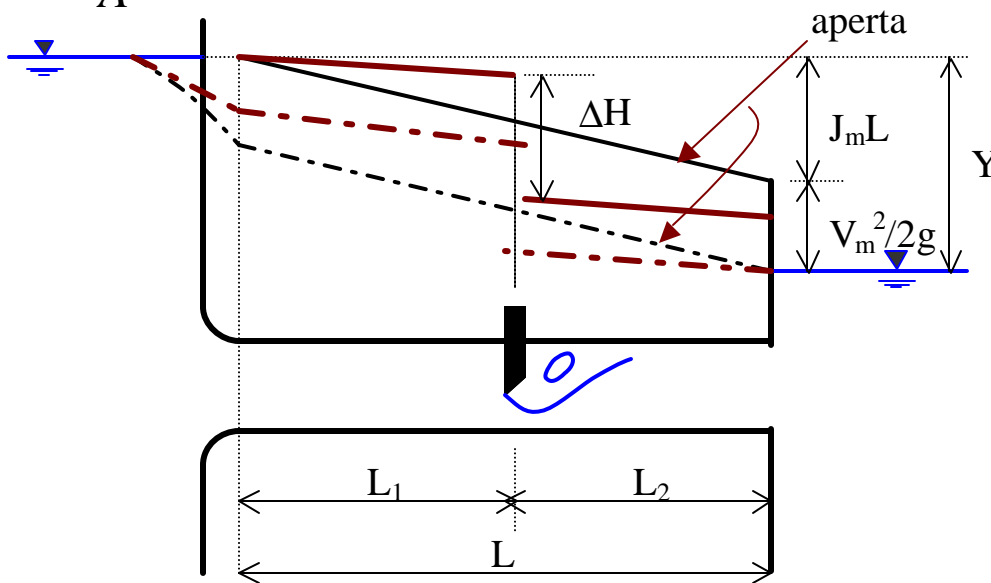
# STROZZAMENTI

## Cambiamenti di direzione



$$\Delta H = \frac{(V_c - V)^2}{2 \cdot g} = \frac{V^2}{2 \cdot g} \cdot \left( \frac{1}{m \cdot C_c} - 1 \right)^2$$

$$m = \frac{A_1}{A}$$

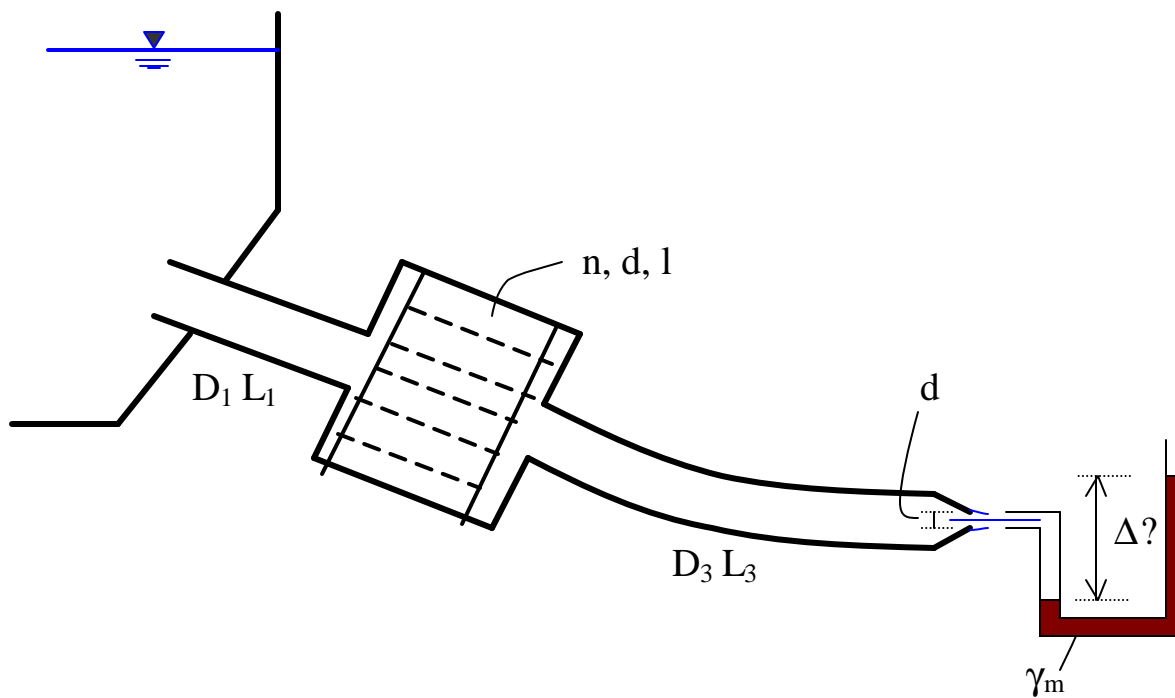


$$Y = L_1 \cdot J + L_2 \cdot J + \Delta H + \frac{V^2}{2 \cdot g}$$

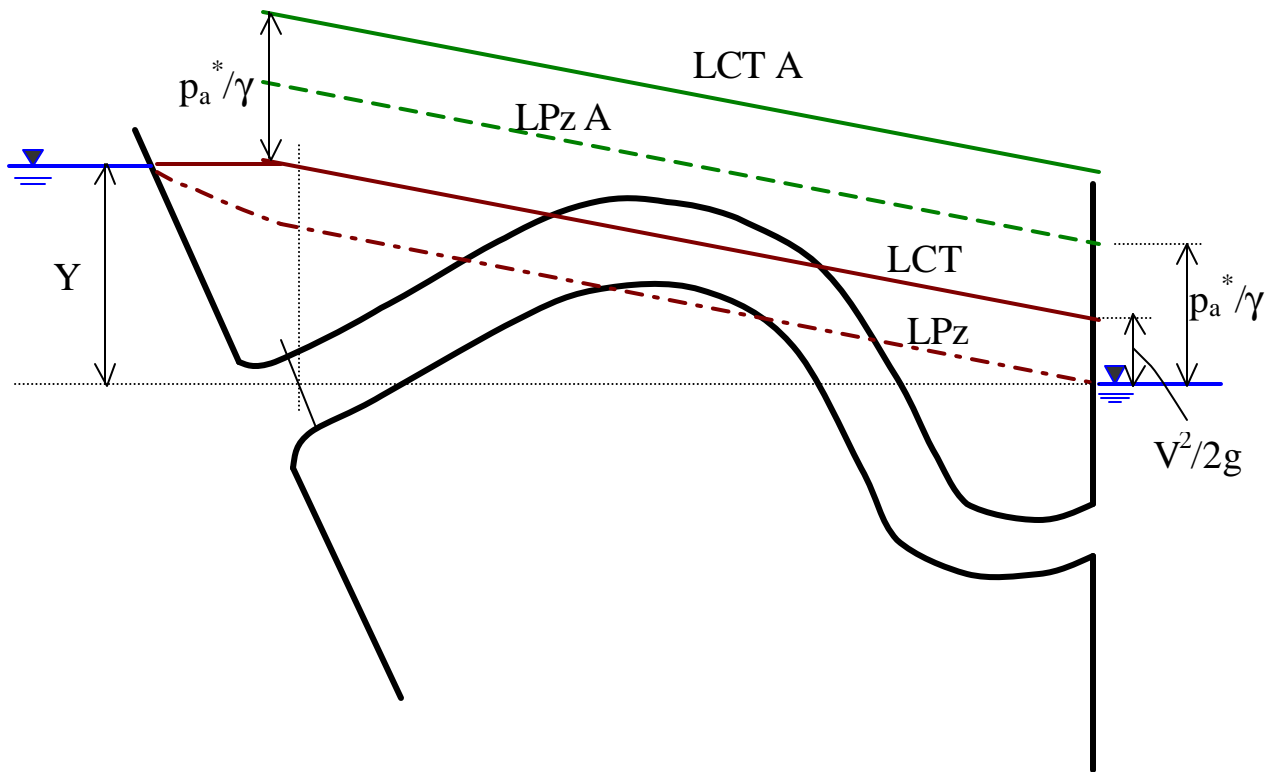
$$Y = \frac{V^2}{2 \cdot g} \cdot \left[ I \cdot \frac{L}{D} + \left( \frac{1}{m \cdot C_c} - 1 \right)^2 + 1 \right]$$

$m \downarrow V \downarrow$

# ESEMPIO

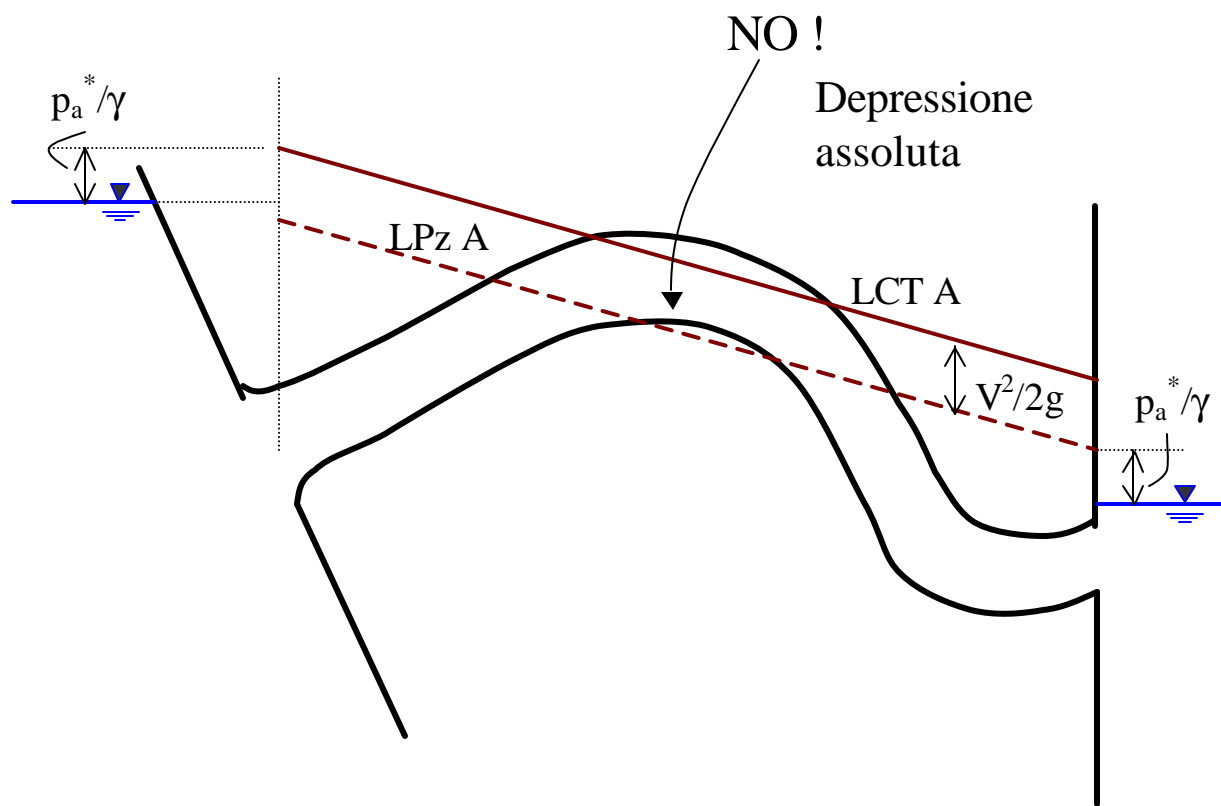


# CONDOTTE IN DEPRESSIONE

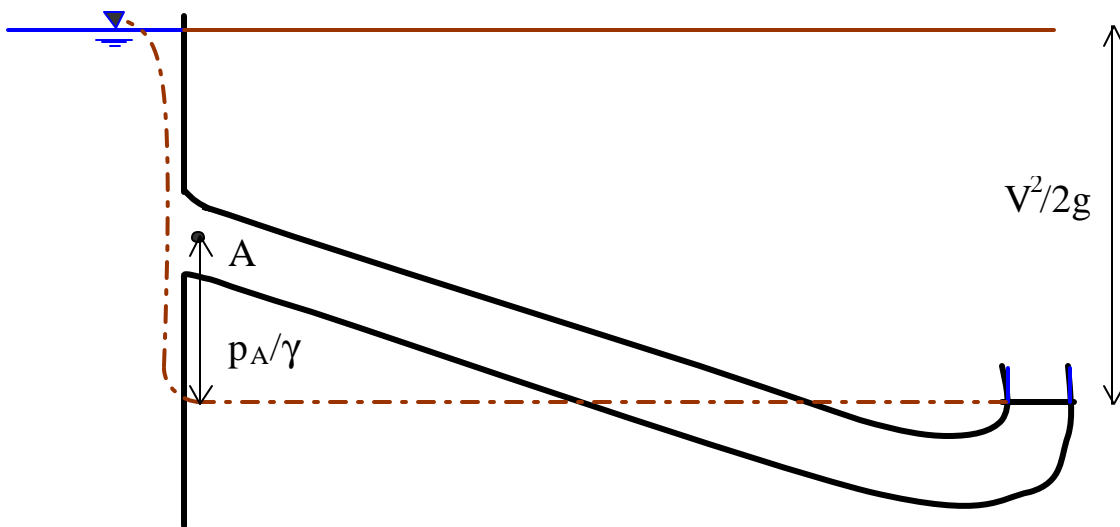


$$\frac{V^2}{2g} + J L = Y$$

$$J = 1 \frac{V^2}{2g D} = \frac{V^2}{C^2 R}$$



# FLUIDO PERFETTO



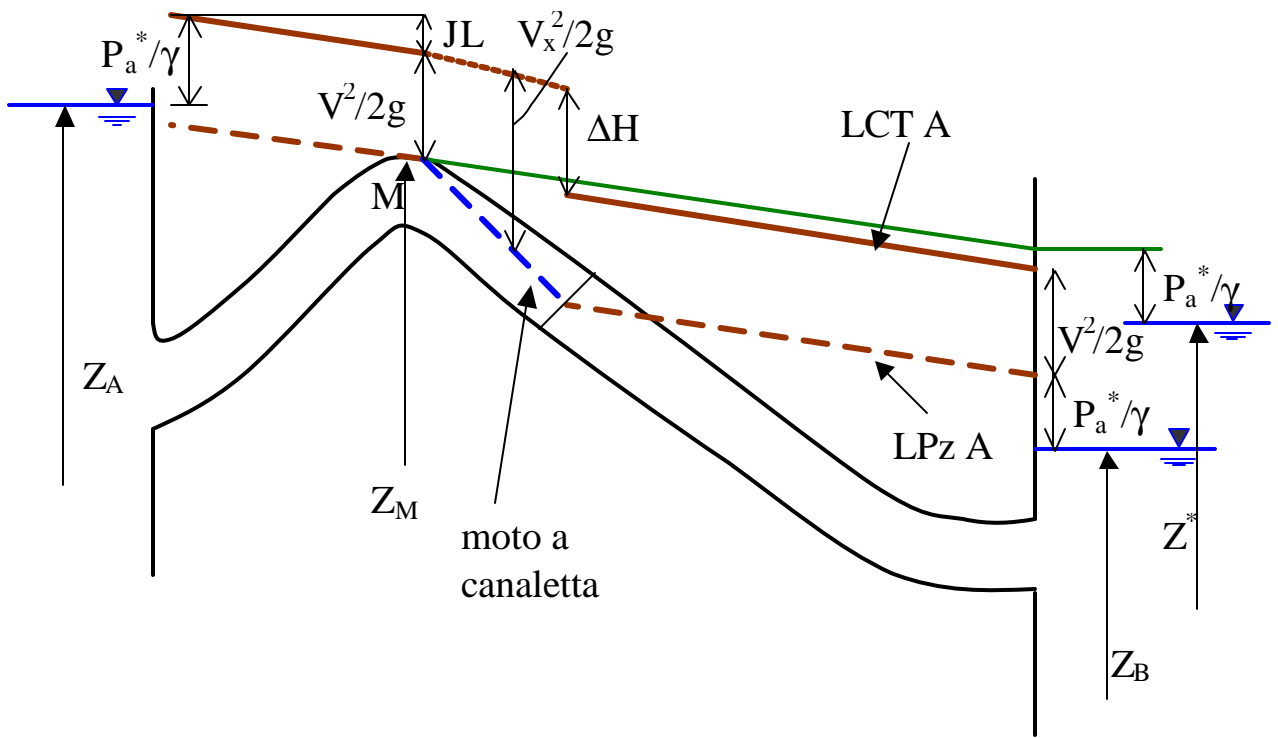
Caso 1

Se in tutti i punti della condotta è verificata la relazione:

$$\left| \frac{p}{\rho g} \right| \leq \frac{p_a^*}{\rho g}$$

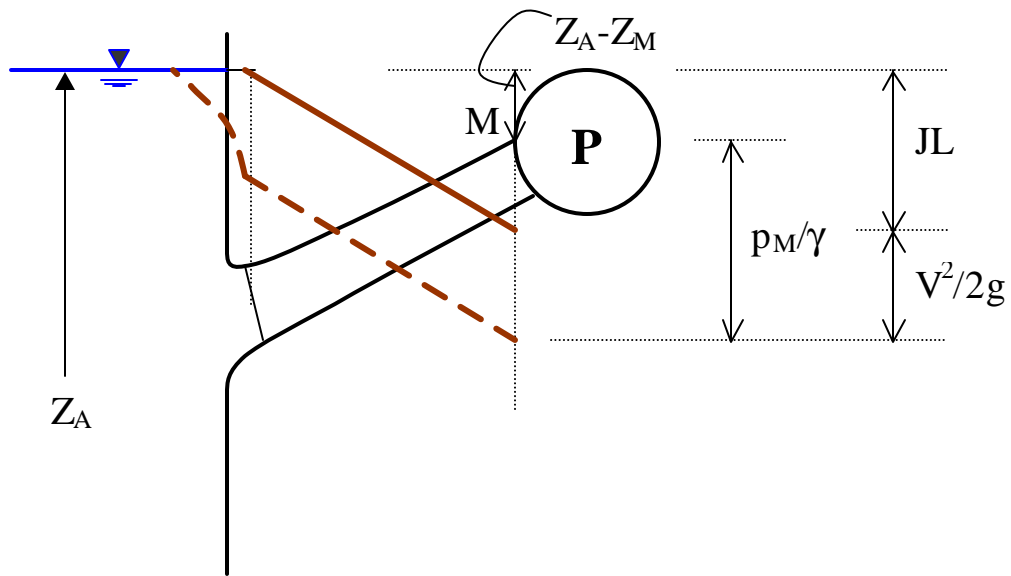
*il moto è REGOLARE e la situazione è quella rappresentata in figura.*





$$Z_A + \frac{p_a^*}{g} = Z_M + \frac{p_M^*}{g} + \frac{V_{\max}^2}{2 \cdot g} + J \cdot L' \quad \left( J = \frac{V_{\max}^2}{c^2 \cdot R} \right)$$

$$Z_A = Z^* + \frac{V_{\max}^2}{2 \cdot g} + L \cdot \frac{V_{\max}^2}{c^2 \cdot R} \Rightarrow Z^*$$



## Depressione

$$\frac{P_M}{g} = J \cdot L + \frac{V^2}{2 \cdot g} - (Z_A - Z_M) \leq \frac{P_a^*}{g} \quad (7 \div 8m)$$